Probabilistic Data Association in Information Space for Generic Sensor Data Fusion

Probabilistische Datenassoziation im Informationsraum zur generischen Sensordatenfusion

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Motivation

Fusion-System

Prediction

Update

Sensor

Mmmm... Which one today?
**Motivation**

- Probabilistic Data Association (PDA) is a good tool to realize a sensor fusion system\[1\], but …
  - the interface between sensor and fusion system can be different for every kind of sensor.
  - a change of the sensor setup or even a new calibration entails major changes of the fusion system
  - there is still explicit knowledge about the sensor necessary

- **Our goal:** Realizing a fusion system which allows us to regard sensors as „plug & play“ devices.
- **Our approach:** Using the Information Filter in PDA.

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In general, the standard Kalman filter in state space is used for tracking objects:

**Prediction:**

\[
\begin{align*}
\hat{x}_{k|k-1} &= F_{k-1} \hat{x}_{k-1|k-1} + G_{k-1} u_{k-1} \\
P_{k|k-1} &= F_{k-1} P_{k-1|k-1} F^T_{k-1} + Q_{k-1} \\
\hat{z}_{k|k-1} &= H_k \hat{x}_{k|k-1} \\
S_k &= H_k P_{k|k-1} H^T_k + R_k
\end{align*}
\]

**Update:**

\[
\begin{align*}
K_k &= P_{k|k-1} H^T_k S_k^{-1} \\
\hat{x}_k &= \hat{x}_{k|k-1} + K_k [z_k - \hat{z}_{k|k-1}] \\
P_k &= P_{k|k-1} - K_k S_k K^T_k
\end{align*}
\]

... and in Information Space

We use the inverse covariance form of the Kalman filter, the information filter:

\[
Y_{k|k}^{-1} = P_{k|k}, \\
\hat{y}_{k|k} = Y_{k|k} \hat{x}_{k|k}
\]

Prediction:

\[
\hat{y}_{k|k-1} = Y_{k|k-1} F_{k-1} Y_{k-1|k-1}^{-1} \hat{y}_{k-1|k-1} \\
Y_{k|k-1} = \left[ F_{k-1} Y_{k-1|k-1}^{-1} F_{k-1}^T + Q_{k-1} \right]^{-1}
\]

Update:

\[
\hat{y}_{k|k} = \hat{y}_{k|k-1} + i_k \\
Y_{k|k} = Y_{k|k-1} + I_k
\]

with

\[
i_k = H_k^T R_k^{-1} z_k \\
I_k = H_k^T R_k^{-1} H_k
\]

Comparison of State and Information Space

Fusion-System

Prediction
$k \rightarrow k - 1$

Kalman Filter

Sensor
$H_k$
$R_k$
$z_k$

Information Filter

Fusion-System

Prediction
$k \rightarrow k - 1$

Sensor
$i_k$
$I_k$

Update

$\hat{x}_{k|k}$ $P_{k|k}$

$\hat{y}_{k|k}$ $Y_{k|k}$

$\hat{x}_{k|k-1}$ $P_{k|k-1}$

$\hat{y}_{k|k-1}$ $Y_{k|k-1}$
PDA in Information Space
PDA Update

\[ P_{k|k} = \sum_{j=0}^{m} \beta_{k}^{x \rightarrow z_{k,j}} \left[ P_{k|k,j} + (\hat{x}_{k|k,j} - \hat{x}_{k|k}) (\hat{x}_{k|k,j} - \hat{x}_{k|k})^T \right] \]  

Update hypothesis for every covariance matrix:

\[ P_{k|k,j} = P_{k|k-1} - K_{k,j} S_{k,j} K_{k,j}^T \]

\[ = \begin{cases} 
P_{k|k-1}, & j = 0 \\
\quad P_{k|k-1} - K_{k,j} S_{k,j} K_{k,j}^T, & j = 1, \ldots, m_k 
\end{cases} \]

instead

Equivalent update hypothesis for every information matrix:

\[ Y_{k|k,j} = Y_{k|k-1} + I_{k,j} \]

\[ = \begin{cases} 
Y_{k|k-1}, & j = 0 \\
Y_{k|k-1} + I_{k,j}, & j = 1, \ldots, m_k 
\end{cases} \]

PDA Update

\[ P_{k|k} = \sum_{j=0}^{m} \beta_{k}^{x \rightarrow z_{k},j} \left[ P_{k|k,j} + (\hat{x}_{k|k,j} - \hat{x}_{k|k}) (\hat{x}_{k|k,j} - \hat{x}_{k|k})^{T} \right] \]

Update hypothesis for every state vector:

\[ \hat{x}_{k|k,j} = \begin{cases} \hat{x}_{k|k-1}, & j = 0 \\ \hat{x}_{k|k-1} + K_{k,j} \nu_{k,j}, & j = 1, \ldots, m_{k} \end{cases} \]

instead

Equivalent update hypothesis for every information vector:

\[ \hat{y}_{k|k,j} = \begin{cases} \hat{y}_{k|k-1}, & j = 0 \\ \hat{y}_{k|k-1} + i_{k,j}, & j = 1, \ldots, m_{k} \end{cases} \]
With these equivalent update hypotheses, the update can now be done using information vector and matrix.

\[ P_{k|k} = \sum_{j=0}^{m} \beta_k^{x \rightarrow z_{k,j}} \left[ P_{k|k,j} + (\hat{x}_{k|k,j} - \hat{x}_{k|k}) (\hat{x}_{k|k,j} - \hat{x}_{k|k})^T \right] \]

\[ \hat{x}_{k|k} = \sum_{j=0}^{m} \beta_k^{x \rightarrow z_{k,j}} \hat{x}_{k|k,j} \]

\[ \hat{y}_{k|k,j} \]

\[ \hat{x}_{k|k,j} = P_{k|k,j} \hat{y}_{k|k,j} \]

\[ Y_{k|k,j}^{-1} \]

\[ P_{k|k,j} = Y_{k|k,j}^{-1} \]
The Association Weights

To calculate the complete update, the association weights are necessary.

\[ \beta_{k}^{x \rightarrow z_{k,j}} = \begin{cases} \frac{b}{b + \sum_{l=1}^{m_{k}} e_{k,l}}, & j = 0 \\ \frac{e_{k,j}}{b + \sum_{l=1}^{m_{k}} e_{k,l}}, & j = 1, \ldots, m_{k} \end{cases} \]

\[ e_{k,j} = e^{-\frac{1}{2} \nu_{k,j}^{T} S_{k,j}^{-1} \nu_{k,j}} \]

\[ S = HPH^{T} + R \]

\[ \nu = z - \hat{z} \]

the exponent can be expressed as \([3]:\)

\[ \nu_{k,j}^{T} \chi_{k,j}^{\dagger} \nu_{k,j} = \nu_{k,j}^{T} S_{k,j}^{-1} \nu_{k,j} \]

with

\[ \nu_{k,j} = i_{k,j} - I_{k,j} \hat{x}_{k|k-1} \]

\[ \chi_{k,j} = I_{k,j} + I_{k,j} P_{k|k-1} I_{k,j} \]

\[ \nu_{k,j} \] measurement residuum

\[ \chi_{k,j} \] pseudo-inverse

The Association Weights

To calculate the complete update, the association weights are necessary.

\[ \beta_{k}^{x ightarrow z_{k,j}} = \begin{cases} \frac{b}{b + \sum_{l=1}^{m_{k}} e_{k,l}}, & j = 0 \\ \frac{e_{k,j}}{b + \sum_{l=1}^{m_{k}} e_{k,l}}, & j = 1, \ldots, m_{k} \end{cases} \]

are necessary.

\[ b = \left( \frac{2 \pi}{\gamma} \right)^{\frac{n_{z}}{2}} \lambda V_{k} c_{n_{z}} \frac{(1 - P_{D} P_{G})}{P_{D}}, \text{ parametric} \]

Using the parametric model, \(|S_{k,tj}|\) is needed to approximate the gating volume \(V_{k}\). Therefore we derived the approximation:

\[ |S_{k,tj}| = \frac{\|\Upsilon_{k,j} + \|H_{k,j}\|^{2}}{\|I_{k,j}\|^{2}} + \text{pseudo-determinant} \]

\(\lambda\) mean clutter density
\(c_{n_{z}}\) volume unity sphere
\(P_{G}\) gating probability
\(P_{D}\) detection probability
JIPDA in Information Space
Joint Integrated Probabilistic Data Association (JIPDA)

The calculation of the association weights makes the most important algorithmic difference of PDA and J(I)PDA. They can be calculated using a tree based approach\(^5\).

\[\text{true positive (TP)} \quad \text{track } t \text{ generated measurement } j\]

\[\text{false positive (FP)} \quad \text{measurement } j \text{ is clutter}\]

\[\text{false negative (FN)} \quad \text{object } t \text{ exists, but was not detected}\]

\[\text{true negative (TN)} \quad \text{object } t \text{ was not detected, because it does not exist}\]

\[\text{birth candidate (BC)} \quad \text{measurement } j \text{ is a birth candidate}\]

The True Positive Likelihood

The True Positive Likelihood

\[
p(e = (t, j)) = \Lambda_{t,j} \cdot p_t^D \cdot p_t^F \cdot (1 - p_j^F) \]

\[
\Lambda_{t,j} = \frac{1}{P_G} \cdot \mathcal{N}(z_{k,j} | \hat{z}_{k,t}, S_{k,t,j})
\]

\[
\mathcal{N}(z_{k,j} | \hat{z}_{k,t}, S_{k,t,j}) = \int \mathcal{N}(z_{k,j} | H_{k,j} x_{k,t}, R_{k,j}) \mathcal{N}(x_{k,t} | \hat{x}_{k,t}, P_{k,t}) dx_{k,t}
\]

with

\[
\mathcal{N}(z_{k,j} | H_{k,j} x_{k,t}, R_{k,j}) = c \cdot \mathcal{N}(i_{k,j} | I_{k,j} x_{k,t}, I_{k,j})
\]

it follows

\[
\mathcal{N}(z_{k,j} | \hat{z}_{k,t}, S_{k,t,j}) = c \cdot \mathcal{N}(i_{k,j} | I_{k,j} x_{k,t}, I_{k,j} \hat{P}_k | k_t I_{k,j}^T + I_{k,j})
\]

measurement likelihood

detection probability

\[p_t^D\]

pred. existence probability

\[p_t^F\]

sensory clutter probability

\[p_j^F\]
Comparison of State and Information Space

State Space

Information Space

Fusion-System

Prediction

Update

JIPDA

Sensor

\( H_k \ R_k \)

\( z_k \ dim(z) \)

Fusion-System

JIPDA

Sensor

\( i_k \ I_k \)

\( \begin{vmatrix} H_{k,j} \\ R_{k,j} \end{vmatrix} \)
Results
Results

• Evaluation in three different scenarios using the optimal subpattern assignment (OSPA) metric\(^6\).

• Two scenarios with simulated data
  1. Linear measurement model
  2. Non-linear measurement model

• One scenario with real data
  • Linear measurement model

Track switches are penalized using the Track-OSPA\(^7\) (\(\alpha\) parameter). The cutoff is \(c = 10\).

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Results for simulated data (non-linear)

Trajectories of the simulated data scenario.
Results for real data (linear)
Results for real data (linear)

Trajectories of the pedestrians in the real data scenario
Conclusion

• The information filter approach was integrated into the probabilistic data association.

• The new method can be used with linear and non-linear measurement models.
  • In case of a non-linear measurement model further analysis is required because of the possible singularities.

• The use of the Probabilistic Data Association in Information Space allows a generic fusion framework independent of any sensor.

• Thus no knowledge about the sensor principle is necessary.
Thank you for your attention

*Essentially, all models are wrong, but some are useful.*

George E. P. Box