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KO-FAS

Probabilistic Data Association in Information Space for Generic Sensor Data Fusion

Probabilistische Datenassoziation im Informationsraum zur generischen Sensordatenfusion

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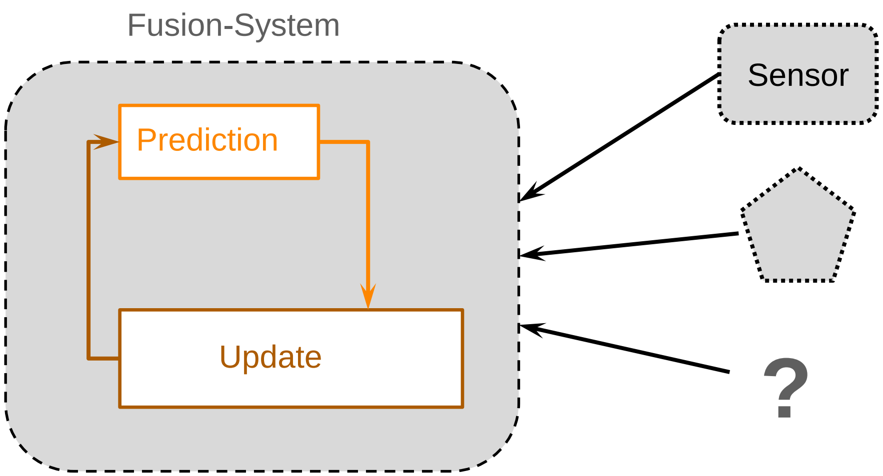
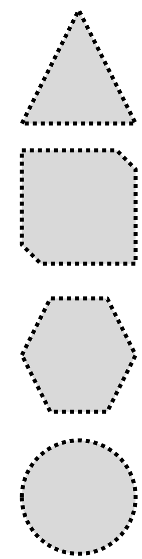
Motivation



Fusion-System
Prediction
Update
Sensor
?

MMMM...
WHICH ONE
TODAY ?



Motivation

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- Probabilistic Data Association (PDA) is a good tool to realize a sensor fusion system[1] , but …
  - the interface between sensor and fusion system can be different for every kind of sensor.
  - a change of the sensor setup or even a new calibration entails major changes of the fusion system
  - there is still explicit knowledge about the sensor necessary

- Our goal: Realizing a fusion system which allows us to regard sensors as „plug & play“ devices.
- Our approach: Using the Information Filter in PDA.

[1] Munz et al. (2009). Proc. of the 12th Int. IEEE Conf. on Int. Transp. Sys, St. Louis, MO, USA.

The Kalman Equations in State Space, …

In general, the standard Kalman filter in state space is used for tracking objects:

Prediction:

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_{k-1}\hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^{\mathrm{T}} + \boldsymbol{Q}_{k-1}$$

$$\hat{\boldsymbol{z}}_{k|k-1} = \boldsymbol{H}_k\hat{\boldsymbol{x}}_{k|k-1}$$

$$\boldsymbol{S}_k = \boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_k$$

Update:

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{S}_k^{-1}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k\left[\boldsymbol{z}_k - \hat{\boldsymbol{z}}_{k|k-1}\right]$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\boldsymbol{S}_k\boldsymbol{K}_k^{\mathrm{T}}$$

[2] Kalman (1960). Transactions of the ASME- Journal of Basic Engineering, pp. 35-45.

… and in Information Space

We use the inverse covariance form of the Kalman filter, the information filter:

$$\boldsymbol{Y}_{k|k}^{-1} = \boldsymbol{P}_{k|k},$$
$$\hat{\boldsymbol{y}}_{k|k} = \boldsymbol{Y}_{k|k}\hat{\boldsymbol{x}}_{k|k}$$

Prediction:

$$\hat{\boldsymbol{y}}_{k|k-1} = \boldsymbol{Y}_{k|k-1}\boldsymbol{F}_{k-1}\boldsymbol{Y}_{k-1|k-1}^{-1}\hat{\boldsymbol{y}}_{k-1|k-1}$$
$$\boldsymbol{Y}_{k|k-1} = \left[\boldsymbol{F}_{k-1}\boldsymbol{Y}_{k-1|k-1}^{-1}\boldsymbol{F}_{k-1}^{T} + \boldsymbol{Q}_{k-1}\right]^{-1}$$

Update:

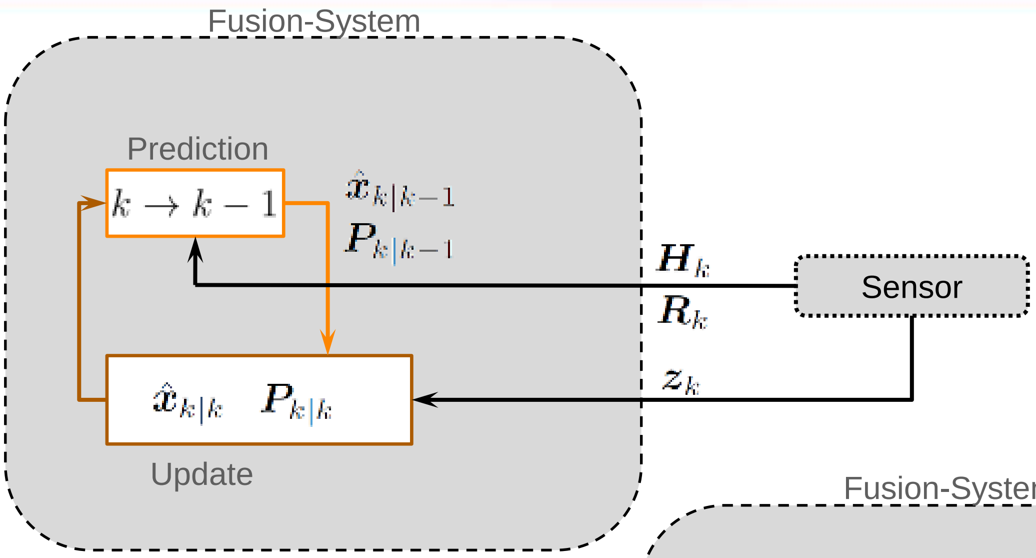
$$\hat{\boldsymbol{y}}_{k|k} = \hat{\boldsymbol{y}}_{k|k-1} + \boldsymbol{i}_k$$
$$\boldsymbol{Y}_{k|k} = \boldsymbol{Y}_{k|k-1} + \boldsymbol{I}_k$$

with

$$\boldsymbol{i}_k = \boldsymbol{H}_k^T\boldsymbol{R}_k^{-1}\boldsymbol{z}_k$$
$$\boldsymbol{I}_k = \boldsymbol{H}_k^T\boldsymbol{R}_k^{-1}\boldsymbol{H}_k$$

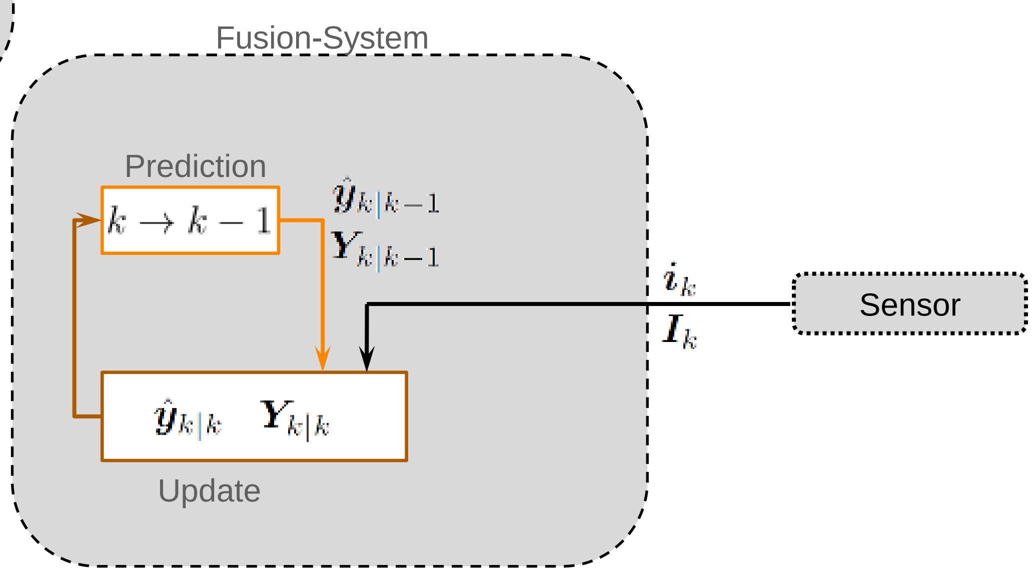
[3] Grime et al. (1992). American Control Conference, 1992, pp. 3299-3303.

Comparison of State and Information Space



Kalman Filter

Information Filter



PDA in Information Space

PDA Update

$$\boldsymbol{P}_{k|k} = \sum_{j=0}^{m} \beta_k^{\boldsymbol{x} \to \boldsymbol{z}_{k,j}} \left[\boldsymbol{P}_{k|k,j} + \left(\hat{\boldsymbol{x}}_{k|k,j} - \hat{\boldsymbol{x}}_{k|k} \right) \left(\hat{\boldsymbol{x}}_{k|k,j} - \hat{\boldsymbol{x}}_{k|k} \right)^{\mathrm{T}} \right]$$ [4]

Update hypothesis for every **covariance** matrix:

$$\begin{aligned} \boldsymbol{P}_{k|k,j} &= \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_{k,j} \boldsymbol{S}_{k,j} \boldsymbol{K}_{k,j}^{\mathrm{T}} \\ &= \begin{cases} \boldsymbol{P}_{k|k-1}, & j = 0 \\ \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_{k,j} \boldsymbol{S}_{k,j} \boldsymbol{K}_{k,j}^{\mathrm{T}}, & j = 1, \ldots, m_k \end{cases} \end{aligned}$$

↓ instead

Equivalent update hypothesis for every **information** matrix:

$$\begin{aligned} \boldsymbol{Y}_{k|k,j} &= \boldsymbol{Y}_{k|k-1} + \boldsymbol{I}_{k,j} \\ &= \begin{cases} \boldsymbol{Y}_{k|k-1}, & j = 0 \\ \boldsymbol{Y}_{k|k-1} + \boldsymbol{I}_{k,j}, & j = 1, \ldots, m_k \end{cases} \end{aligned}$$

[4] Bar-Shalom et al. (1975). Automatica 11, pages 451-460.

PDA Update

$$\boldsymbol{P}_{k|k} = \sum_{j=0}^{m} \beta_k^{\boldsymbol{x} \to \boldsymbol{z}_{k,j}} \left[\boldsymbol{P}_{k|k,j} + \left(\hat{\boldsymbol{x}}_{k|k,j} - \hat{\boldsymbol{x}}_{k|k} \right) \left(\hat{\boldsymbol{x}}_{k|k,j} - \hat{\boldsymbol{x}}_{k|k} \right)^{\mathrm{T}} \right]$$

Update hypothesis for every **state** vector:

$$\hat{\boldsymbol{x}}_{k|k,j} = \begin{cases} \hat{\boldsymbol{x}}_{k|k-1}, & j = 0 \\ \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_{k,j} \boldsymbol{\nu}_{k,j}, & j = 1, \ldots, m_k \end{cases}$$

instead

Equivalent update hypothesis for every **information** vector:

$$\begin{aligned} \hat{\boldsymbol{y}}_{k|k,j} &= \hat{\boldsymbol{y}}_{k|k-1} + \boldsymbol{i}_{k,j} \\ &= \begin{cases} \hat{\boldsymbol{y}}_{k|k-1}, & j = 0 \\ \hat{\boldsymbol{y}}_{k|k-1} + \boldsymbol{i}_{k,j}, & j = 1, \ldots, m_k \end{cases} \end{aligned}$$

PDA Update

$$\boldsymbol{P}_{k|k} = \sum_{j=0}^{m} \beta_k^{\boldsymbol{x}\rightarrow \boldsymbol{z}_{k,j}} \left[\boldsymbol{P}_{k|k,j} + \left(\hat{\boldsymbol{x}}_{k|k,j} - \hat{\boldsymbol{x}}_{k|k}\right)\left(\hat{\boldsymbol{x}}_{k|k,j} - \hat{\boldsymbol{x}}_{k|k}\right)^{\mathrm{T}}\right]$$

$$\hat{\boldsymbol{x}}_{k|k} = \sum_{j=0}^{m} \beta_k^{\boldsymbol{x}\rightarrow \boldsymbol{z}_{k,j}} \hat{\boldsymbol{x}}_{k|k,j}$$

$$\hat{\boldsymbol{y}}_{k|k,j}$$

$$\hat{\boldsymbol{x}}_{k|k,j} = \boldsymbol{P}_{k|k,j}\hat{\boldsymbol{y}}_{k|k,j}$$

$$\boldsymbol{Y}_{k|k,j}^{-1}$$

$$\boldsymbol{P}_{k|k,j} = \boldsymbol{Y}_{k|k,j}^{-1}$$

➡ With these equivalent update hypotheses, the update can now be done using information vector and matrix.

The Association Weights

To calculate the complete update, the association weights

$$\beta_k^{\boldsymbol{x} \to \boldsymbol{z}_{k,j}} = \begin{cases} \frac{b}{b + \sum_{l=1}^{m_k} e_{k,l}}, & j = 0 \\ \frac{e_{k,j}}{b + \sum_{l=1}^{m_k} e_{k,l}}, & j = 1, \ldots, m_k \end{cases}$$

are necessary.

$$e_{k,j} = \mathrm{e}^{-\frac{1}{2} \boldsymbol{\nu}_{k,j}^{\mathrm{T}} \boldsymbol{S}_{k,j}^{-1} \boldsymbol{\nu}_{k,j}}$$

$\boldsymbol{S} = \boldsymbol{H} \boldsymbol{P} \boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R}$

$\boldsymbol{\nu} = \boldsymbol{z} - \hat{\boldsymbol{z}}$

→ the exponent can be expressed as [3]:

$$\boldsymbol{v}_{k,j}^T \boldsymbol{\Upsilon}_{k,j}^{\dagger} \boldsymbol{v}_{k,j} = \boldsymbol{\nu}_{k,j}^T \boldsymbol{S}_{k,j}^{-1} \boldsymbol{\nu}_{k,j}$$

with

$$\boldsymbol{v}_{k,j} = \boldsymbol{i}_{k,j} - \boldsymbol{I}_{k,j} \hat{\boldsymbol{x}}_{k|k-1}$$

$$\boldsymbol{\Upsilon}_{k,j} = \boldsymbol{I}_{k,j} + \boldsymbol{I}_{k,j} \boldsymbol{P}_{k|k-1} \boldsymbol{I}_{k,j}$$

$\boldsymbol{\nu}_{k,j}$ measurement residuum

$\cdot^{\dagger}$ pseudo-inverse

[3] Grime et al. (1992). American Control Conference, 1992, pp. 3299-3303.

The Association Weights

To calculate the complete update, the association weights

$$\beta_k^{\boldsymbol{x}\rightarrow\boldsymbol{z}_{k,j}} = \begin{cases} \frac{b}{b+\sum_{l=1}^{m_k} e_{k,l}}, & j=0 \\ \frac{e_{k,j}}{b+\sum_{l=1}^{m_k} e_{k,l}}, & j=1,\ldots,m_k \end{cases}$$

are necessary.

$$b = \left(\frac{2\pi}{\gamma}\right)^{\frac{n_z}{2}} \lambda V_k c_{n_z} \frac{(1-P_D P_G)}{P_D}$$, parametric

Using the parametric model, $|\boldsymbol{S}_{k,tj}|$ is needed to approximate the gating volume V_k. Therefore we derived the approximation:

$$|\boldsymbol{S}_{k,tj}| = \frac{|\boldsymbol{\Upsilon}_{k,j}|_+ |\boldsymbol{H}_{k,j}|_+^2}{|\boldsymbol{I}_{k,j}|_+^2}$$

- λ mean clutter density
- c_{n_z} volume unity sphere
- P_G gating probability
- P_D detection probability
- $|\cdot|_+$ pseudo-determinant

JIPDA in Information Space

Joint Integrated Probabilistic Data Association (JIPDA)

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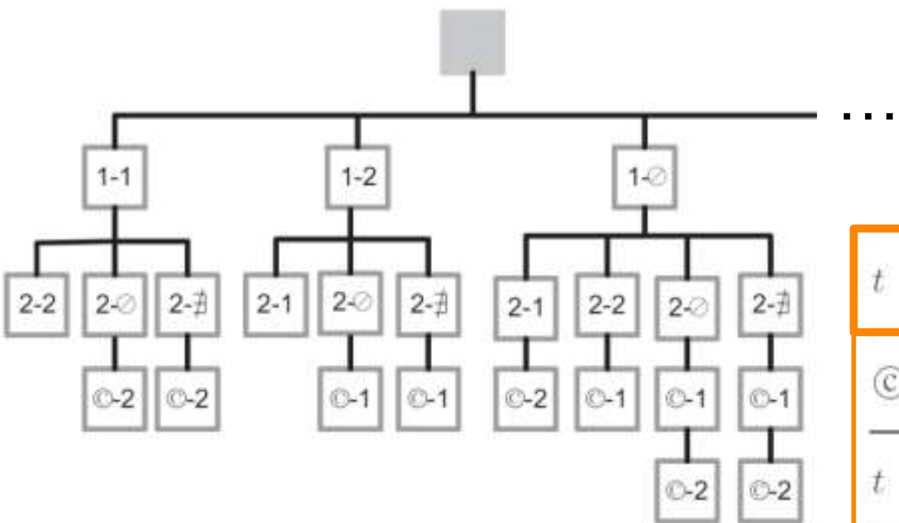
The calculation of the association weights makes the most important algorithmic difference of PDA and J(I)PDA. They can be calculated using a tree based approach[5] .



$t \rightarrow j$	true positive (TP) track t generated measurement j
$© \rightarrow j$	false positive (FP) measurement j is clutter
$t \rightarrow \emptyset$	false negative (FN) object t exists, but was not detected
$t \rightarrow \nexists$	true negative (TN) object t was not detected, because it does not exist
$b \rightarrow j$	birth candidate (BC) measurement j is a birth candidate

[5] Mählisch et al. (2008). Proc. 5th. Int. Workshop on Intelligent Transportation, pp. 1-6.

The True Positive Likelihood

$t \to j$	true positive (TP) track t generated measurement j

$$p(e = (t, j)) = \Lambda_{t,j} \cdot p_t^D \cdot p_t^\exists \cdot (1 - p_j^F)$$

$$\boldsymbol{\Lambda}_{t,j} = \frac{1}{P_G} \cdot \mathcal{N}(\boldsymbol{z}_{k,j} | \hat{\boldsymbol{z}}_{k,t}, \boldsymbol{S}_{k,tj})$$

$$\mathcal{N}(\boldsymbol{z}_{k,j} | \hat{\boldsymbol{z}}_{k,t}, \boldsymbol{S}_{k,tj}) = \int \mathcal{N}(\boldsymbol{z}_{k,j} | \boldsymbol{H}_{k,j} \boldsymbol{x}_{k,t}, \boldsymbol{R}_{k,j}) \mathcal{N}(\boldsymbol{x}_{k,t} | \hat{\boldsymbol{x}}_{k,t}, \boldsymbol{P}_{k,t}) \mathrm{d}\boldsymbol{x}_{k,t}$$

with

$$\mathcal{N}(\boldsymbol{z}_{k,j} | \boldsymbol{H}_{k,j} \boldsymbol{x}_{k,t}, \boldsymbol{R}_{k,j}) = c \cdot \mathcal{N}(\boldsymbol{i}_{k,j} | \boldsymbol{I}_{k,j} \boldsymbol{x}_{k,t}, \boldsymbol{I}_{k,j})$$

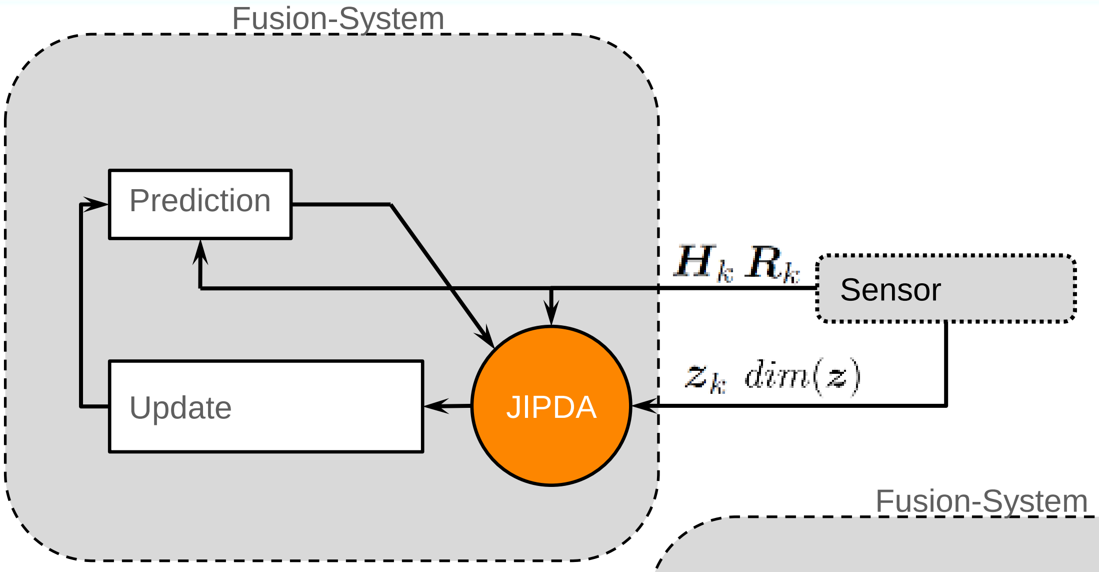
it follows

$$\mathcal{N}(\boldsymbol{z}_{k,j} | \hat{\boldsymbol{z}}_{k,t}, \boldsymbol{S}_{k,tj}) = c \cdot \mathcal{N}(\boldsymbol{i}_{k,j} | \boldsymbol{I}_{k,j} \boldsymbol{x}_{k,t}, \boldsymbol{I}_{k,j} \hat{\boldsymbol{P}}_{k|k,t} \boldsymbol{I}_{k,j}^{\mathrm{T}} + \boldsymbol{I}_{k,j})$$

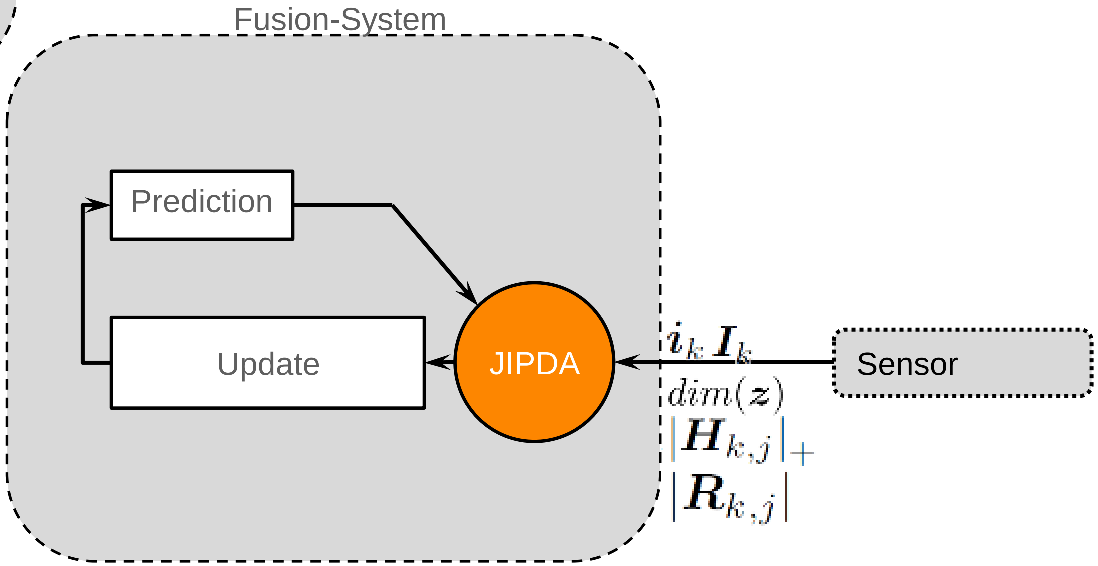
- $\Lambda_{t,j}$ measurement likelihood
- p_t^D detection probability
- $p_t^\exists$ pred. existence probability
- p_j^F sensory clutter probability

→ With our new formulation of the measurement likelihood, the weights of the update can be calculated.

Comparison of State and Information Space



State Space



Information Space

Results

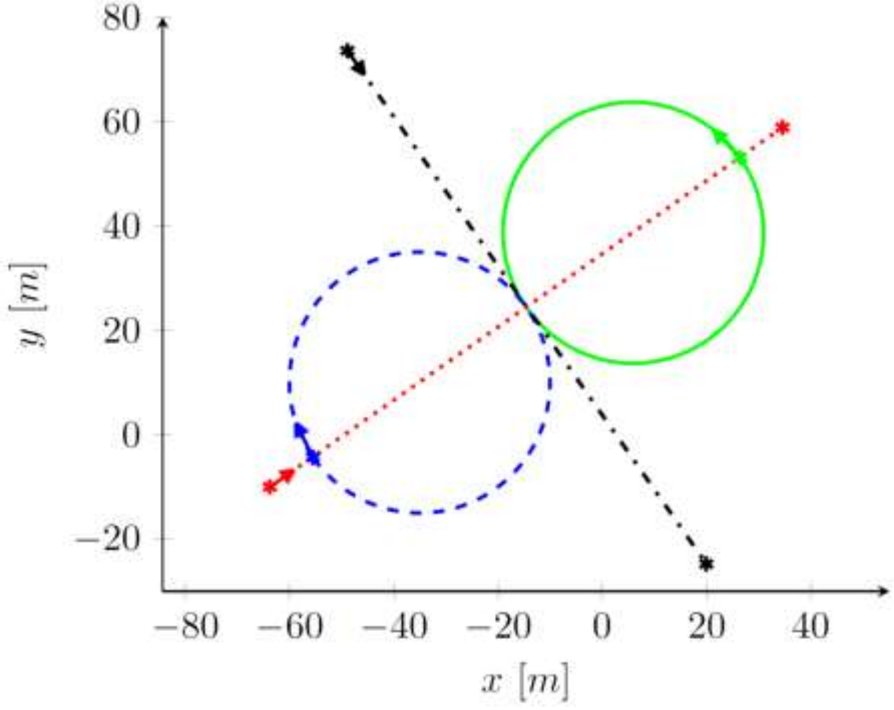
Results

- Evaluation in three different scenarios using the optimal subpattern assignment (OSPA) metric[6].
- Two scenarios with simulated data
  1. Linear measurement model
  2. Non-linear measurement model
- One scenario with real data
  - Linear measurement model

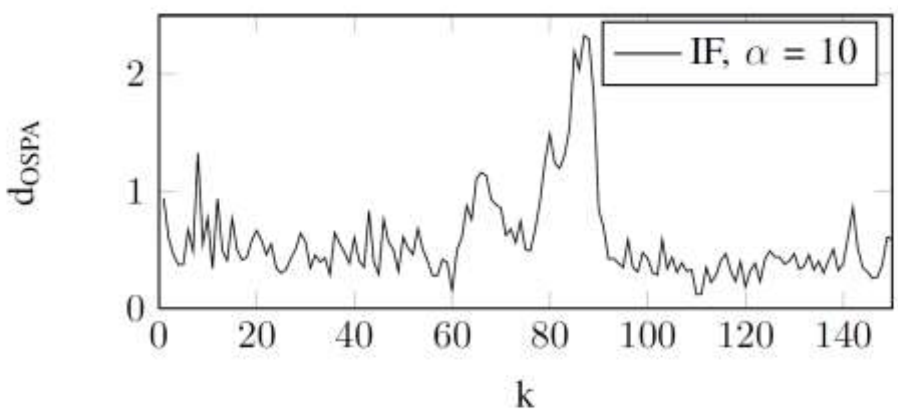
Track switches are penalized using the Track-OSPA[7] (α parameter). The cutoff is $c = 10$.

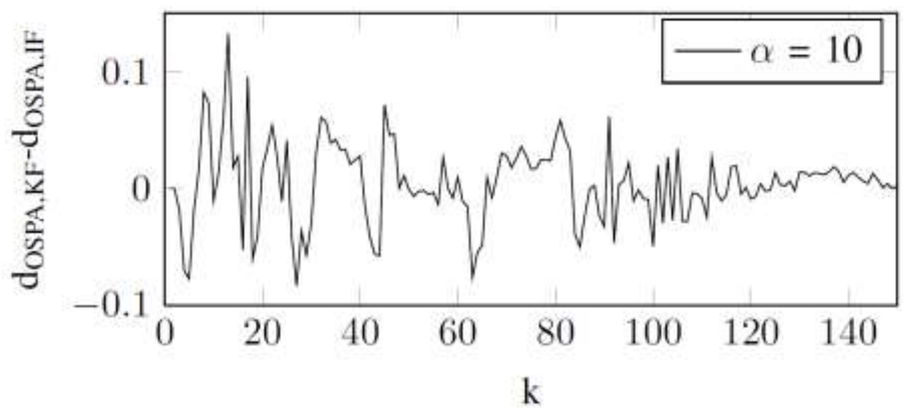
[6] Schuhmacher et al. (2008). IEEE Transactions on Signal Processing, vol. 56, no.8, pp. 3447-3457.
[7] Ristic et al. (2009). IEEE Transactions on Signal Processing, vol. 59, no.7, pp. 3452-3457.

Results for simulated data (non-linear)

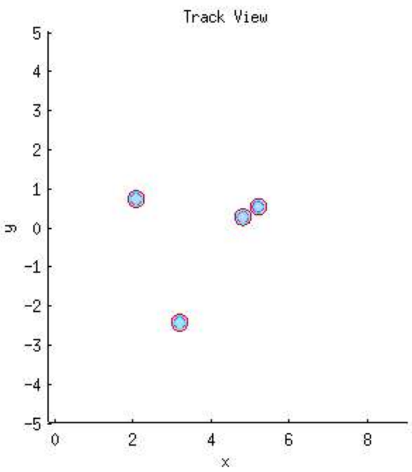


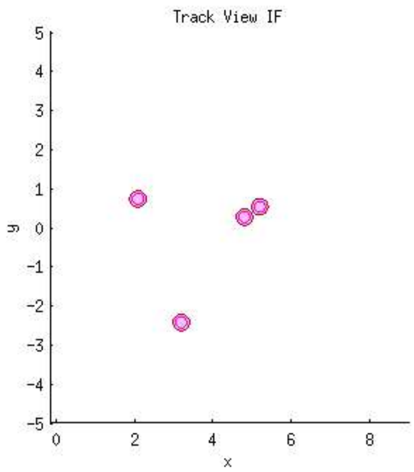
Trajectories of the simulated data scenario.

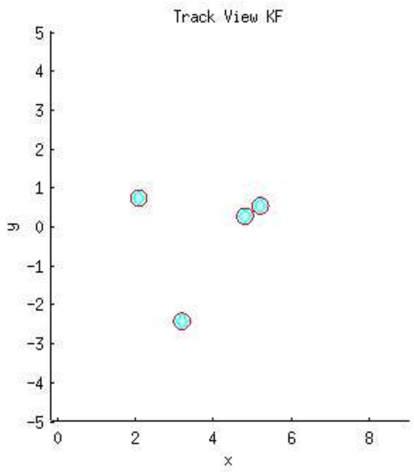




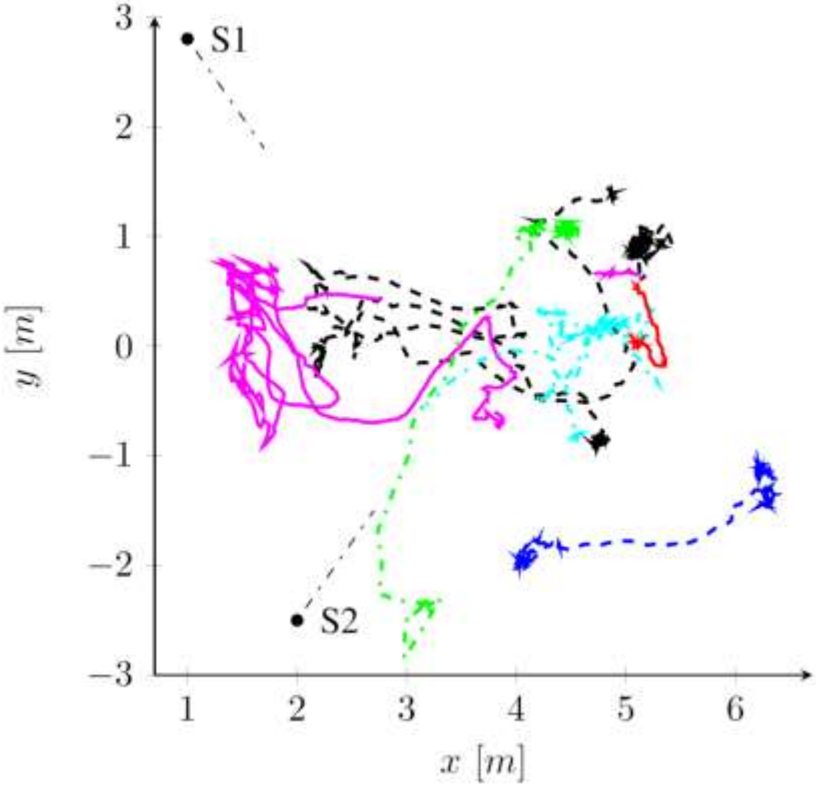
Results for real data (linear)



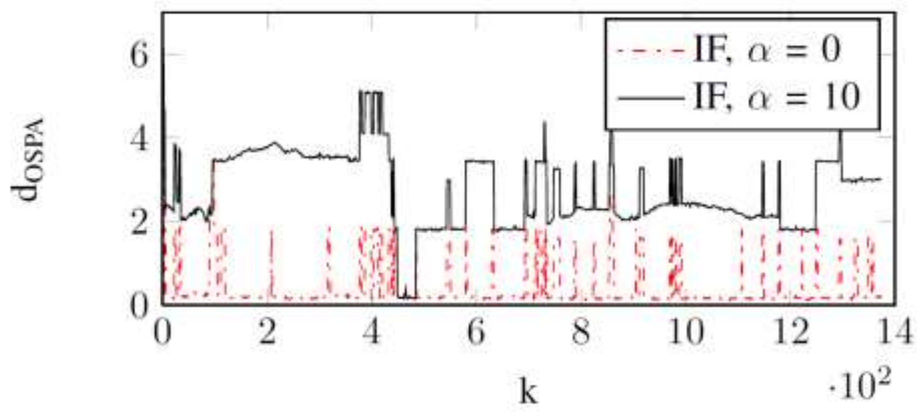


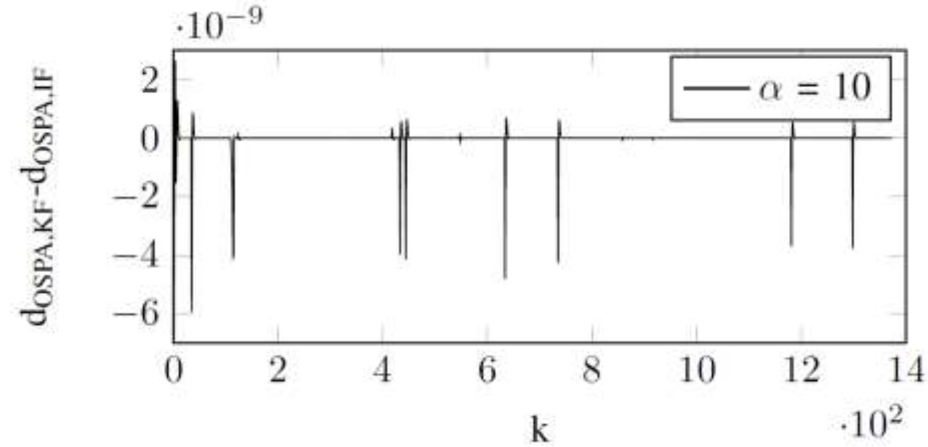


Results for real data (linear)



Trajectories of the pedestrians in the real data scenario





Conclusion

- The information filter approach was integrated into the probabilistic data association.
- The new method can be used with linear and non-linear measurement models.
  - In case of a non-linear measurement model further analysis is required because of the possible singularities.
- The use of the Probabilistic Data Association in Information Space allows a generic fusion framework independent of any sensor.
- Thus no knowledge about the sensor principle is necessary.

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Thank you for your attention

Essentially, all models are wrong, but some are useful.

George E. P. Box